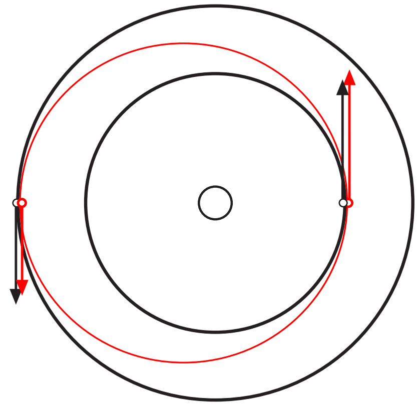
Finding Low delta V paths to NEOs

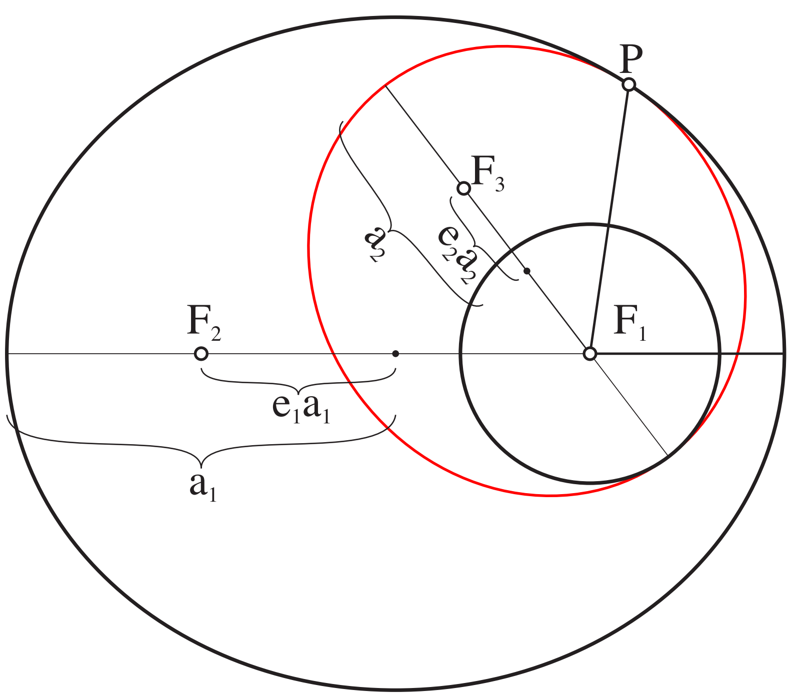


When a transfer orbit is tangent to coplanar departure and destination orbits, velocity vectors point in the same direction.

No delta V is needed for direction change, only speed change.

A transfer orbit tangent to two coplanar, circular orbits is the well known Hohmann transfer orbit.

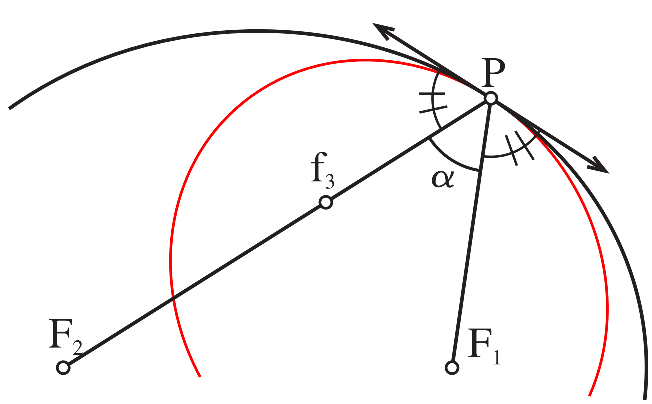
But what if the destination orbit is an ellipse? For destination orbits, we'll look at coplanar asteroid orbits with perihelion > 1 A.U. Like the Hohmann transfer orbit, we want our transfer orbits to be tangent to the destination as well as the departure orbit. This way no delta V is needed for direction change.



Call the asteroid's orbit **Ellipse 1**.
Ellipse 1's eccentricity: e_1
semi major axis: a_1
sun's center: F_1
2nd focus: F_2

Call the red transfer orbit **Ellipse 2**.
eccentricity: e_2
semi major axis: a_2
sun's center: F_1
2nd focus: F_3

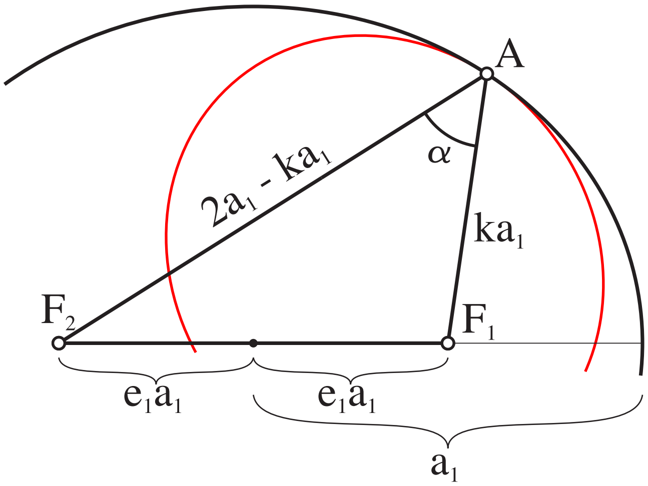
Call the point of tangency P



Ellipses 1 and 2 share a tangent line at point P. A ray emanating from a focus will be reflected to the second focus with angle of incidence equal to angle of reflection.

This implies P, F_3, and F_2 are collinear.

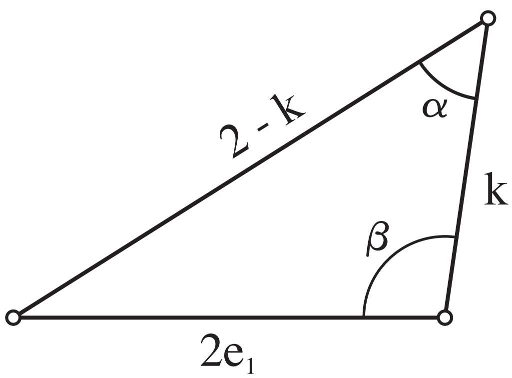
Also angle F_1PF_2 = angle F_1PF_3. Call this angle α .



The distance between F_1 and P we call ka_1.

$1 - e_1 \leq k \leq 1 + e_1$

The other sides of the triangle can be deduced by properties of an ellipse.



Given a specific value for k, we'd like to know angle α.

We can use the triangle to the left since it's similar to triangle F_1PF_2.

Law of cosines gives us

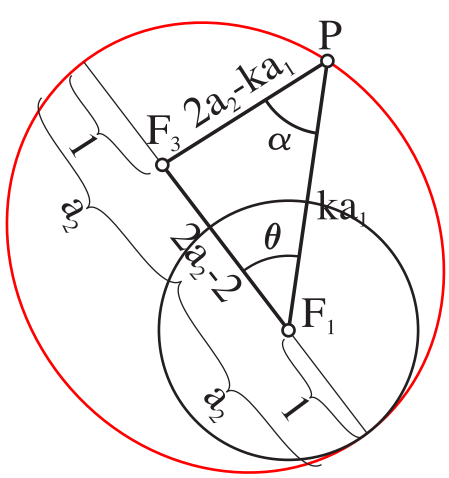
$k^2 + (2-k)^2 - 2k(2-k)\cos\alpha = (2e_1)^2$

$\alpha = \text{acos}(((2-2e_1^{\ 2})/(k(2-k))) - 1)$

Also, $\beta = \text{acos}(e_1/k - 1/ke_1 + 1/e_1)$

So if know the asteroid orbit's eccentricity we can determine angles alpha and beta for a given value of k.

Now we look at the transfer orbit, ellipse 2.



The transfer ellipse is tangent to earth's orbit so we know it's perihelion is 1 A.U.

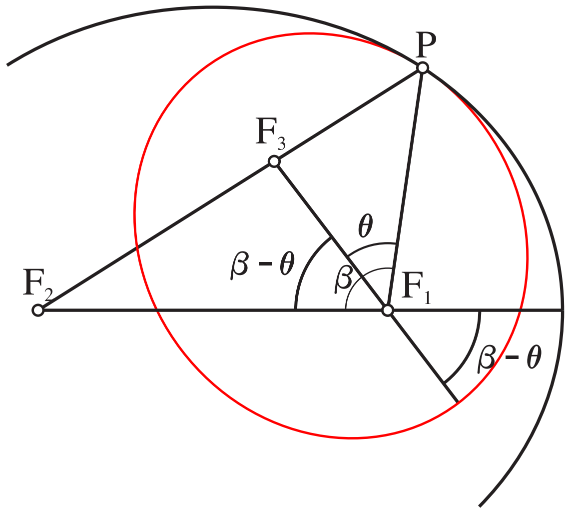
Therefore the distance between f_1 and f_3 is $2a_2$-2.

The third side can be inferred from a property of the ellipse.

Using the law of cosines we can deduce

$a_2 = (2 - k^2a_1^{\ 2}(1+\cos\alpha)) / (-2ka_1(1+\cos\alpha)+4)$

$\theta = \text{acos}((ka_1a_2-2a_2+1) / (ka_1a_2 -ka_1))$

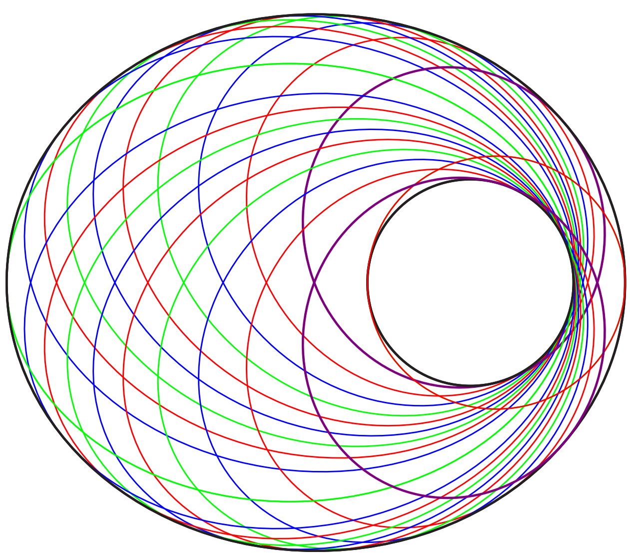


Ellipse 2's axis is at an angle of $\beta - \theta$ to Ellipse 1's axis.

Since opposite angles are equal, we can see the transfer orbit's longitude of periapsis ($\tilde{\omega}_2$) differs from the destination orbit's longitude of periapsis ($\tilde{\omega}_1$) by an angle of $\beta - \theta$.

If A lies after perihelion and before aphelion $(\tilde{\omega}_2) = (\tilde{\omega}_1) - (\beta - \theta)$

If A lies before perihelion and after aphelion $(\tilde{\omega}_2) = (\tilde{\omega}_1) + (\beta - \theta)$



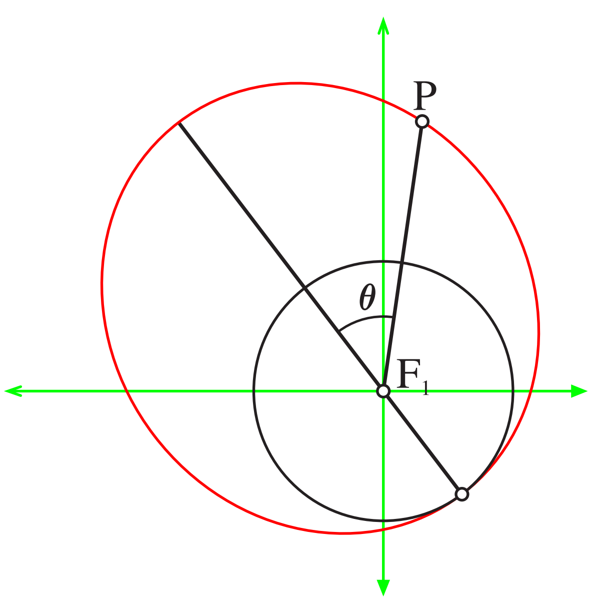
We set up a spread sheet to do increments of k from $1-e_1$ to $1+e_1$.

This gives us a spectrum of possible tangent transfer orbits.

For each transfer orbit we can find a, e, and $\tilde{\omega}$.

Since we're assuming coplanar orbits, i and line of nodes are undefined.

How about τ, time of periapsis?



Recall we leave earth at the transfer orbit's perihelion. That the longitude of periapsis coincides with earth's position indicates the date.

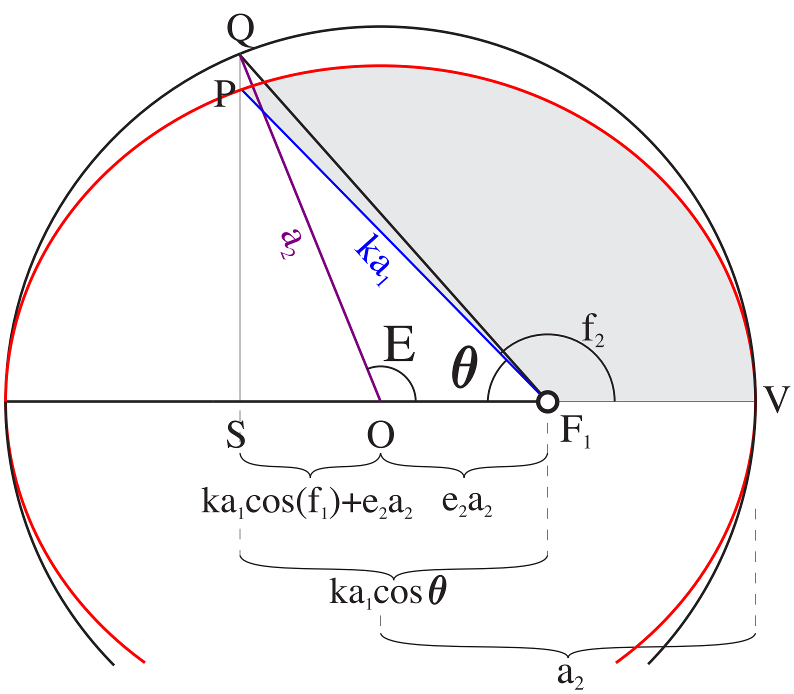
For example, a 0 degree longitude of periapsis implies departure during the autumnal equinox, around September 21.

In the illustration to the left, departure from earth occurs at 307 degrees which would be around July 30.

We would like to know the date the transfer orbit arrives at it’s destination.

As the transfer vehicle moves between earth and the destination orbit, it sweeps out the grey wedge below. What fraction is this of Ellipse 2?

Vertically scaling Ellipse 2 by $(1- e_2^2)^{-1/2}$ gives a circle having radius a_2.



If P is after perihelion and before aphelion,
True anomaly $f_2 = \pi - \theta$
if P is afer perihelion or
$\pi + \theta$ if P is after aphelion.

$OS = a_2 \cos E = a_2 e_2 + k a_1 \cos f_2$

Radius vector is ka_1. The polar equation for an ellipse gives:

$$ka_1 = a_2(1 - e_2^2) / (1 + e_2 \cos f_1)$$

Substituting for ka_1:

$$a_2 \cos E = \frac{a_2 e_2 + a_2(1 - e_2^2)\cos f_2}{1 + e_2 \cos f_2}$$

$$\cos E = \frac{e_2 + \cos f_2}{1 + e_2 \cos f_2}$$

Area triangle F_1OQ is $e_2 a_2^2 \sin E / 2$. Area wedge OQV is $E\, a_2^2 / 2$.

Area FQV = Area Wedge OQV - Area triangle F_1OQ

$$\text{Area FQV} = E\, a_2^2 / 2 - e_2 a_2^2 \sin E / 2$$

$$= \frac{a_2^2}{2}(E - e_2 \sin E)$$

Using A.U., and years as units, a transfer orbit’s period is $a_2^{3/2}$ years.

Time between earth departure and P is $a_2^{3/2}(E - e_2 \sin E) / (2\pi)$

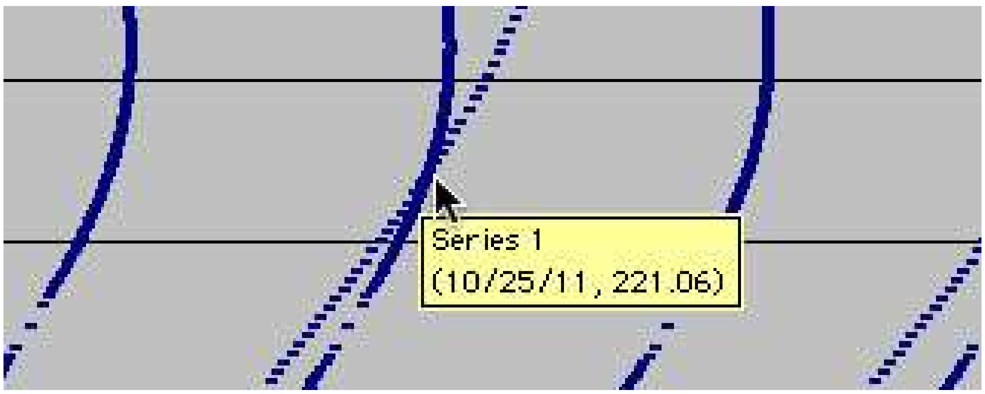
By adding trip times to departure dates, we can get dates of arrival.

We now have arrival dates and longitudes for the tangent transfer orbits.

But how often will the target NEO be at the right place at the right time?

To determine this we graph the true longitude (L) of the tangent transfer arrivals by date as well as the asteroid's true longitude by date.

Where the paths on the graph intersect, we have a time and place for where the asteroid is amenable to receiving a tangent transfer orbit. In Excel, dragging the cusor over a location on the graph gives information on the point:



Scrolling through the various increments of k, I found the transfer ellipse that arrives at the true longitude of 221 degrees at October 25. From this row I can also get departure date and longitude as well as other information.

My term "ka_1" is another term for "r".

We know the semi-major axis a the transfer orbit as well as the destination orbit. So we can get the speed for both objects at rendezvous using the vis-viva equation:

$$v = 2\pi\sqrt{2/r - 1/a}$$

Since I use A.U., years, and solar mass as my units, μ becomes $4\pi^2$.

I convert A.U./Year to km/sec by multiplying by 4.74.

Recall that the transfer orbit is tangent to the destination orbit, so the velocity vectors are pointing in the same direction. So to get $v_{infinity}$, I merely subtract the ship's speed from the asteroid's speed.

We also know r during earth departure so we can figure the ship's speed at that point. We call earth's speed 2πA.U./year. To get $v_{infinity}$ at departure, we subtract the ship's speed from earth's speed.

To our spreadsheet we added $v_{infinity}$ at departure as well as arrival.